



Flow and heat transfer inside thin films supported by soft seals in the presence of internal and external pressure pulsations

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Received 27 April 2002; received in revised form 14 June 2002

Abstract

The effects of both external squeezing and internal pressure pulsations are studied on flow and heat transfer inside non-isothermal and incompressible thin films supported by soft seals. The laminar governing equations are non-dimensionalized and reduced to simpler forms. The upper plate displacement is related to the internal pressure through the elastic behavior of the supporting seals. The following parameters: squeezing number, squeezing frequency, frequency of pulsations, Fixation number (for the seal) and the thermal squeezing parameter are found to be the main controlling parameters. Accordingly, their influences on flow and heat transfer inside disturbed thin films are determined and discussed. It is found that an increase in the Fixation number results in more cooling and a decrease in the average temperature values. Also, it is found that an increase in the squeezing number decreases the turbulence level at the upper plate. Furthermore, fluctuations in the heat transfer and the fluid temperatures can be maximized at relatively lower frequency of internal pressure pulsations.

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Keywords: Thin films; Heat transfer; Squeezing; Pressure pulsations; Seals

1. Introduction

Various engineering applications such as lubrication, heat pipes and fluidic cells of many chemical and biological detection systems require deeper understanding of flow and heat transfer inside thin films. In certain applications, external disturbances such as unbalances in rotating machines or pulsations in external ambient pressures due to many disturbances can result in an oscillatory motion at the boundary. Not only external disturbances can produce these motions, but also internal pressure pulsations such as irregularities in the pumping process can produce similar effects. Even small disturbances on the plates of the thin film can have a

substantial impact as the thickness of thin films is very small. This fact is more pronounced if the thin film is supported by soft seals. Accordingly, the dynamics and thermal characterization of thin films will be altered.

The chambers for chemical and biological detection systems such as fluidic cells for chemical or biological microcantilever probes [1] are an important example for thin films. Small turbulence levels that can be introduced into these cells by either flow pulsating at the inlet or external noise that may be present at the boundaries which result in a vibrating boundary can produce flow instabilities inside the fluidic cells. These disturbances have a substantial influence on the measurements of the biological probes specially those utilizing microcantilevers as these detecting elements are very sensitive to flow conditions.

Several authors have considered flow inside squeezed thin films like Langlois [2] who performed an analytical study for flow inside isothermal oscillatory squeezed

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Nomenclature

B	thin film length	Y, y	dimensionless and dimensional normal coordinates
c_p	specific heat of the fluid	<i>Greek symbols</i>	
d_s	characteristic dimension of the seal	α	thermal diffusivity for the fluid
E	modulus of elasticity for the seal's material	β, β_p	dimensionless squeezing motion and pressure pulsation amplitudes
F_n	Fixation number	ε	perturbation parameter
H, h, h_0	dimensionless, dimensional and reference thin film thickness	γ, γ_p	dimensionless squeezing motion and pressure pulsation frequencies
h_c	convective heat transfer coefficient	μ	dynamic viscosity of the fluid
k	thermal conductivity of the fluid	θ, θ_m	dimensionless temperature and dimensionless mean bulk temperature
Nu_L, Nu_U	lower and upper plates Nusselt numbers	θ_w	dimensionless temperature at the lower plate (UHF)
P_S	thermal squeezing parameter	ρ	density of the fluid
p	fluid pressure	τ	dimensionless time
q	reference heat flux at the lower plate for UHF	σ	squeezing number
T, T_1	temperature in fluid and the inlet temperature	ω	reciprocal of a reference time (reference squeezing frequency)
T_2	temperature at the lower and the upper plates for CWT	η	variable transformation for the dimensionless Y -coordinate
t	time	Θ	dimensionless heat transfer parameter (CWT)
V_0	reference axial velocity	Π	dimensionless pressure
U, u	dimensionless and dimensional axial velocities	Π_1, Π_0	dimensionless inlet pressure and dimensionless mean pressure
V, v	dimensionless and dimensional normal velocities		
X, x	dimensionless and dimensional axial coordinates		

films with fluid density varying according to the pressure. However, only few of them have analyzed heat transfer inside squeezed thin films such as Hamza [3], Bhattacharyya et al. [4] and Debbaut [5]. In these works, the squeezing was not of oscillatory type. Recently, Khaled and Vafai [6] and [7] considered flow and heat transfer inside incompressible oscillatory squeezed thin films. The effects of internal pressure pulsations have been investigated before on flow and heat transfer inside channels (see [8,9]). However, the literature lacks an investigation of the effects of both internal and external pressure pulsations on flow and heat transfer inside thin films. In this case, the gap thickness will be a function of both pulsations.

In this work, the upper plate of a thin film is considered to be subjected to both external squeezing effects and the internal pressure pulsations. The influence of internal pressure pulsations on the displacement of the upper plate is determined by the theory of linear elasticity applied to the seal supporting the plates of an incompressible non-isothermal thin film. The laminar governing equations for flow and heat transfer are properly non-dimensionalized and reduced into simpler

equations. The resulting equations are then solved numerically to determine the effects of external squeezing, internal pressure pulsations and the strength of the seal on the turbulence inside the disturbed thin films as well as on thermal characteristics of these thin films.

2. Problem formulation

Consider a two dimensional thin film that has a small thickness h compared to its length B . The x -axis is taken in the direction of the length of the thin film while y -axis is taken along the thickness as shown in Fig. 1. The width of the thin film, D , is assumed to be large enough such that two dimensional flow inside the thin film can be assumed. The lower plate of the thin film is fixed while the vertical motion of the upper plate is assumed to have sinusoidal behavior when the thin film gap is not charged with the working fluid. This motion due to only external disturbances is expressed according to the following relation:

$$h = h_0(1 - \beta \cos(\gamma \omega t)) \quad (1)$$

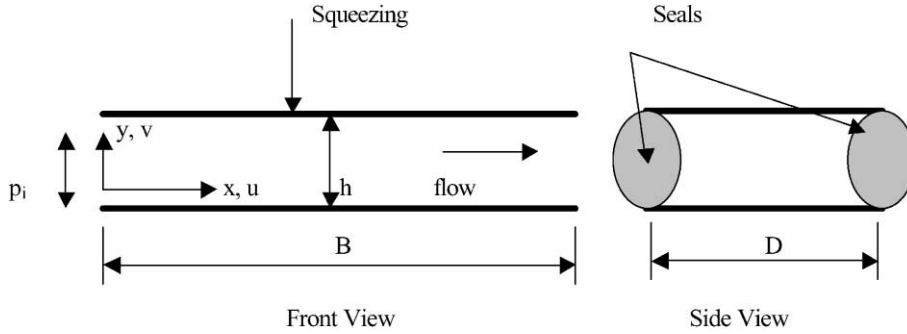


Fig. 1. Schematic diagram for a thin film.

where γ is the dimensionless frequency and β and ω are the dimensionless upper plate motion amplitude and a reference frequency, respectively. It is assumed that the fluid is Newtonian with constant properties.

The general two-dimensional continuity, momentum and energy equations for the laminar thin film are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{3}$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{4}$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{5}$$

where T , ρ , p , μ , c_p and k are the fluid temperature, density, pressure, dynamic viscosity, specific heat and the thermal conductivity of the fluid, respectively.

Eqs. (2)–(5) are non-dimensionalized using the following dimensionless variables:

$$\begin{aligned} X &= \frac{x}{B}, & Y &= \frac{y}{h_0}, & \tau &= \omega t, & U &= \frac{u}{(\omega B + V_0)}, \\ V &= \frac{v}{h_0 \omega}, & \Pi &= \frac{p - p_e}{\mu(\omega + \frac{V_0}{B})\varepsilon^{-2}}, & \theta &= \frac{T - T_1}{\Delta T} \end{aligned} \tag{6}$$

where T_1 and V_0 are the inlet temperature of the fluid and a constant representing a reference dimensional velocity, respectively. ΔT is equal to $T_2 - T_1$ for constant wall temperature (CWT) conditions, T_2 will be the temperature of both lower and upper plates, and it is equal to qh_0/k for uniform wall heat flux (UHF) conditions p_e is a constant representing the exit pressure. The variables X , Y , τ , U , V , Π and θ are the dimensionless forms of x , y , t , u , v , p and T variables, respectively. The above transformations except for dimensionless temperature have been used by Langlois [2] along with the perturbation parameter ε , $\varepsilon = h_0/B$.

Most flows inside thin films are laminar and could be creep flows especially in lubrications and biological applications. Therefore, the low Reynolds numbers flow model is adopted in here. The application of this model to Eqs. (2)–(5) results in the following reduced non-dimensionalized equations:

$$U = \frac{1}{2} \frac{\partial \Pi}{\partial X} (Y - H) \tag{7}$$

$$\frac{\partial U}{\partial X} + \frac{\sigma}{12} \frac{\partial V}{\partial Y} = 0 \tag{8}$$

$$\frac{\partial}{\partial X} \left(H^3 \frac{\partial \Pi}{\partial X} \right) = \sigma \frac{\partial H}{\partial \tau} \tag{9}$$

$$P_s \left(\frac{\partial \theta}{\partial \tau} + \frac{12}{\sigma} U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \varepsilon^2 \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \tag{10}$$

where σ and P_s are called the squeezing number and the thermal squeezing parameter, respectively. They are defined as

$$\sigma = \frac{12}{1 + \frac{V_0}{\omega B}}, \quad P_s = \frac{\rho c_p h_0^2 \omega}{k} \tag{11}$$

The inlet dimensionless pulsating pressure is considered to have the following relation:

$$\Pi_i = \Pi_0 (1 + \beta_p \sin(\gamma_p \omega t + \varphi_p)) \tag{12}$$

where β_p , Π_i and Π_0 are the dimensionless amplitude in the pressure, inlet dimensionless pressure and the mean dimensionless pressure, respectively. The parameters γ_p and φ_p are the dimensionless frequency of the pressure pulsations and a phase shift angle respectively. Due to both pulsations in internal pressure and external disturbances, the dimensionless film thickness H , ($H = h/h_0$), can be represented by the next equation by noting the principle of superposition:

$$H = 1 - \beta \cos(\gamma \omega t) + H_p \tag{13}$$

where H_p is the dimensionless deformation of the seals resulting from pulsations in the internal pressure. It is assumed that the lower plate is fixed and that the upper

plate of the thin film is rigid such that the magnitude of the deformation in the seals is similar to displacement of the upper plate. The dimensionless deformation in the seals due variations in the external pressure is the second term of Eq. (13) on the right. The dimensionless frequency γ is allowed to be different than γ_p .

The dimensionless pressure gradient inside the thin film as a result of the solution to the Reynolds Eq. (9) is

$$\frac{\partial \Pi}{\partial X} = \frac{\sigma}{H^3} \frac{dH}{d\tau} \left(X - \frac{1}{2} \right) - \Pi_0 (1 + \beta_p \sin(\gamma_p \tau + \varphi_p)) \quad (14)$$

The reference velocity, V_0 , that is used to define the dimensionless pressure, axial dimensionless velocity and the squeezing number are taken to be related to the average velocity, u_m , inside the thin film at zero β and β_p and the dimensionless thickness of the thin film that results from the application of the corresponding inlet mean pressure, H_m , through the following relation:

$$V_0 = \frac{u_m}{H_m^2} \quad (15)$$

The previous scaled reference velocity is only function of the mean pressure, viscosity and the reference dimensions of the thin film and it results in the following relation between the inlet mean dimensionless pressure to the squeezing number:

$$\Pi_0 = 12 - \sigma \quad (16)$$

Accordingly, the dimensionless pressure gradient, the dimensionless pressure and the average dimensionless pressure Π_{AVG} inside the thin film are related to the squeezing number through the following equations:

$$\frac{\partial \Pi(X, \tau)}{\partial X} = \frac{\sigma}{H^3} \frac{dH}{d\tau} \left(X - \frac{1}{2} \right) - (12 - \sigma) \times (1 + \beta_p \sin(\gamma_p \tau + \varphi_p)) \quad (17)$$

$$\Pi(X, \tau) = \frac{\sigma}{2H^3} \frac{dH}{d\tau} (X^2 - X) - (12 - \sigma) \times (1 + \beta_p \sin(\gamma_p \tau + \varphi_p))(X - 1) \quad (18)$$

$$\Pi_{AVG}(\tau) = -\frac{\sigma}{12H^3} \frac{dH}{d\tau} + \frac{(12 - \sigma)}{2} (1 + \beta_p \sin(\gamma_p \tau + \varphi_p)) \quad (19)$$

The displacement of the upper plate due internal pressure pulsations is related to the Π_{AVG} through the theory of linear elasticity by the following relation:

$$H_p = F_n \Pi_{AVG} \quad (20)$$

where F_n is equal to

$$F_n = \frac{\mu(V_0 + \omega B)}{E \varepsilon^2 d_s} \quad (21)$$

The parameters E and d_s in the previous equation are the modulus of elasticity of the used seals and a character-

istic dimension for the seal, respectively. The quantity d_s is equal to the effective diameter of the seal's cross section times the ratio of the length of the seals divided by the thin film width. The effective diameter for seals having square cross section is equal to h_0 . The term F_n will be called the Fixation number of the thin film.

The Fixation number F_n represents a ratio between shear stresses inside thin films to the modulus of elasticity of the used seals. Moreover, Eq. (20) is based on the assumption that transient behavior of the seal's deformation is negligible. The values of F_n are of order 0.001–0.1 for long thin films supported by soft seals.

The first set of dimensionless boundary conditions that will be used is for CWTs at both the lower and the upper plates while the second set is by assuming that the lower plate is at UHF conditions and the upper plate is insulated. As such the dimensionless boundary conditions can be written as

CWT

$$\begin{aligned} \theta(X, Y, 0) = 0, \quad \theta(0, Y, \tau) = 0, \quad \theta(X, 0, \tau) = 1 \\ \theta(X, H, \tau) = 1, \quad \frac{\partial}{\partial X} \left(\frac{1 - \theta(1, Y, \tau)}{1 - \theta_m(1, \tau)} \right) = 0 \end{aligned} \quad (22)$$

UHF

$$\begin{aligned} \theta(X, Y, 0) = 0, \quad \theta(0, Y, \tau) = 0, \quad \frac{\partial \theta(X, 0, \tau)}{\partial Y} = -1 \\ \frac{\partial \theta(X, H, \tau)}{\partial Y} = 0, \quad \frac{\partial \theta(1, Y, \tau)}{\partial X} = \frac{\sigma}{12U_m} \left(\frac{1}{P_s H} - \frac{\partial \theta(1, Y, \tau)}{\partial \tau} \right) \end{aligned} \quad (23)$$

The last condition of Eq. (22) is based on the assumption that the flow at the exit of the thin film is thermally fully developed. Moreover, the last thermal condition of Eq. (23) is derived based on an integral energy balance at the exit of the thin film realizing that the axial conduction is negligible at the exit. The calculated thermal parameters that will be considered are the Nusselt numbers at the lower and upper plates, and the dimensionless heat transfer from the upper and lower plates, Θ , for CWT conditions. They are defined according to the following equations:

CWT

$$\begin{aligned} Nu_U(X, \tau) &\equiv \frac{h_c h_0}{k} = \frac{1}{1 - \theta_m(X, \tau)} \frac{\partial \theta(X, H, \tau)}{\partial Y} \\ Nu_L(X, \tau) &\equiv \frac{h_c h_0}{k} = \frac{-1}{1 - \theta_m(X, \tau)} \frac{\partial \theta(X, 0, \tau)}{\partial Y} \\ \Theta(X, \tau) &= \left(\frac{\partial \theta(X, H, \tau)}{\partial Y} - \frac{\partial \theta(X, 0, \tau)}{\partial Y} \right) \end{aligned} \quad (24)$$

UHF

$$Nu_I(X, \tau) \equiv \frac{h_c h_0}{k} = \frac{1}{\theta(X, 0, \tau) - \theta_m(X, \tau)} \quad (25)$$

where θ_m and U_m are the dimensionless mean bulk temperature and the dimensionless average velocity at a given section. They are defined as follows:

$$\theta_m(X, \tau) = \frac{1}{U_m(X, \tau)H} \int_0^H U(X, Y, \tau)\theta(X, Y, \tau) dY$$

$$U_m(X, \tau) = \frac{1}{H} \int_0^H U(X, Y, \tau) dY \tag{26}$$

Due to symmetric flow and thermal conditions for CWT, it is expected that Nusselt numbers at lower and upper plates to be equal.

3. Numerical methods

The dimensionless thickness of the thin film was determined by solving Eqs. (19), (20) and (13) simultaneously. Accordingly, the velocity field, U and V , was determined from Eqs. (7) and (8). The reduced energy equation, Eq. (10), was then solved using the alternative direction implicit (ADI) techniques by transferring the problem to one with constant boundaries using the following transformations: $\tau^* = \tau$, $\xi = X$ and $\eta = Y/H$. Iterative solution was employed for the ξ -sweep of the energy equation for CWT conditions so that both the energy equation and the exit thermal condition, last condition of Eq. (22), are satisfied. The values of 0.008, 0.03, 0.002 were chosen for $\Delta\xi$, $\Delta\eta$ and $\Delta\tau^*$.

4. Discussions of the results

4.1. Effects of pressure pulsations on the dimensionless film thickness

Figs. 2 and 3 describe the importance of the Fixation number F_n on the dimensionless film thickness H and the dimensionless normal velocity at the upper plate $V(X, H, \tau)$, respectively. It is noticed that as F_n increases, H and absolute values of $V(X, H, \tau)$ increase. It is worth noting that Soft fixations have large F_n values. Increases in the viscosity and flow velocities or a decrease in the thin film thickness, perturbation parameter and the seal's modulus of elasticity increase the value of F_n as Eq. (21) predicts.

The effects of pressure pulsations on H are clearly seen for large values of F_n as shown in Figs. 2 and 3. At these values, the frequency of the local maximum or minimum of H is similar to the frequency of the pressure pulsations as seen from Fig. 2. Further, the degree of turbulence at the upper plate is increased when F_n increases as shown in Fig. 3. The fluctuations and the number of local maximum and minimum in $V(X, H, \tau)$

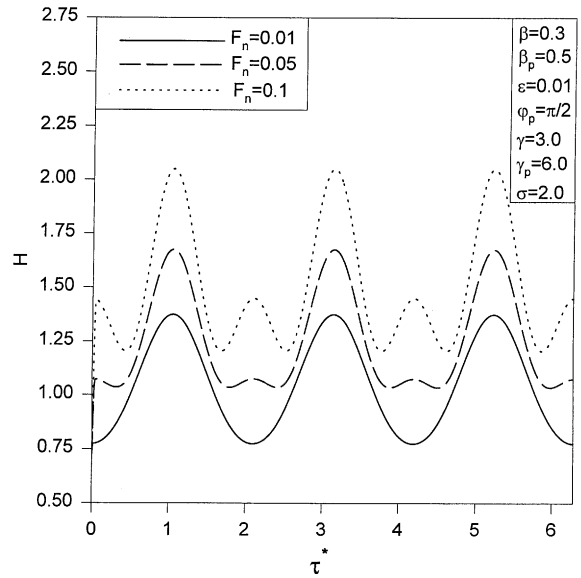


Fig. 2. Effects of F_n on H .

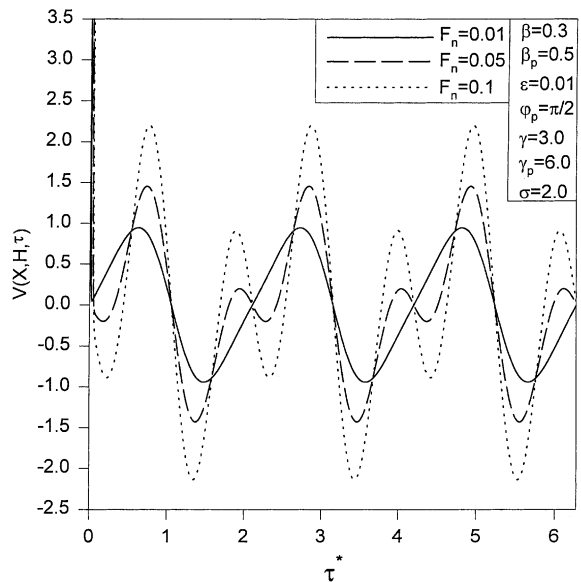


Fig. 3. Effects of F_n on $V(X, H, \tau)$.

are meant by the degree of turbulence at the upper plate. This is also obvious when the values of γ_p increase as shown in Fig. 4. The increase in turbulence level at the upper plate may produce back flows inside the thin film at large values of γ_p . This affects the function of the thin film especially that used as a chamber for detection purposes.

For $\sigma = 12$ where the time average of the average gage pressure inside the thin film is zero, the variation in

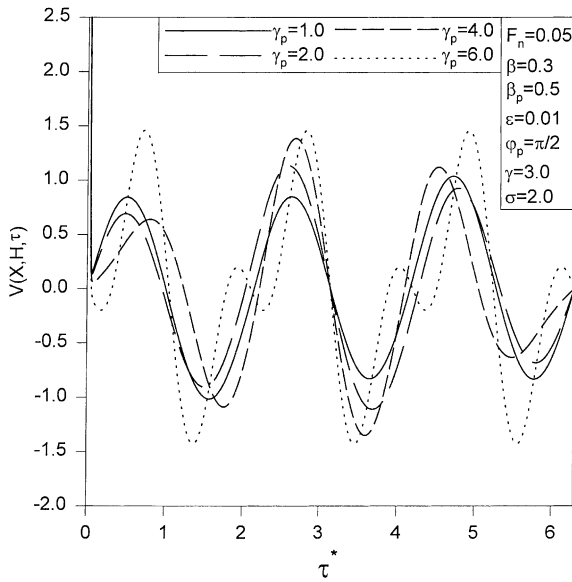


Fig. 4. Effects of γ_p on $V(X, H, \tau)$.

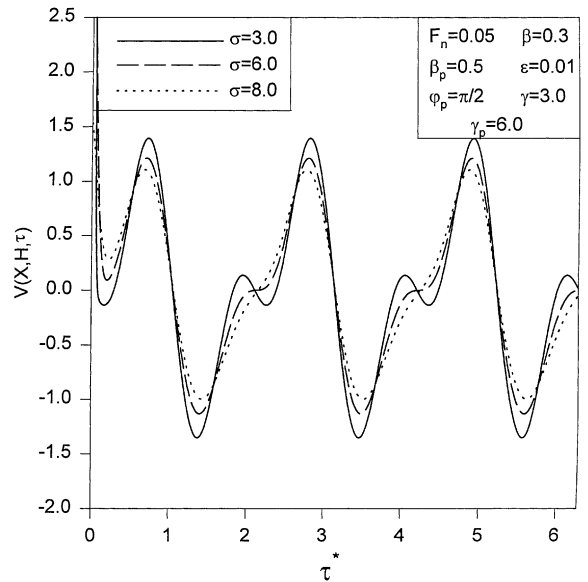


Fig. 6. Effects of σ on $V(X, H, \tau)$.

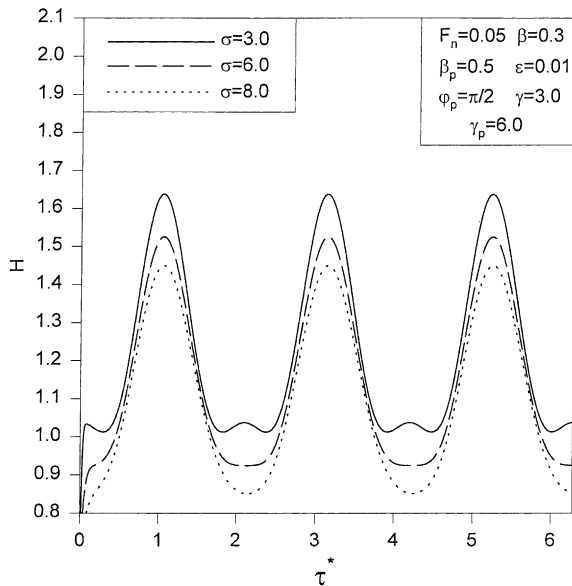


Fig. 5. Effects of σ on H .

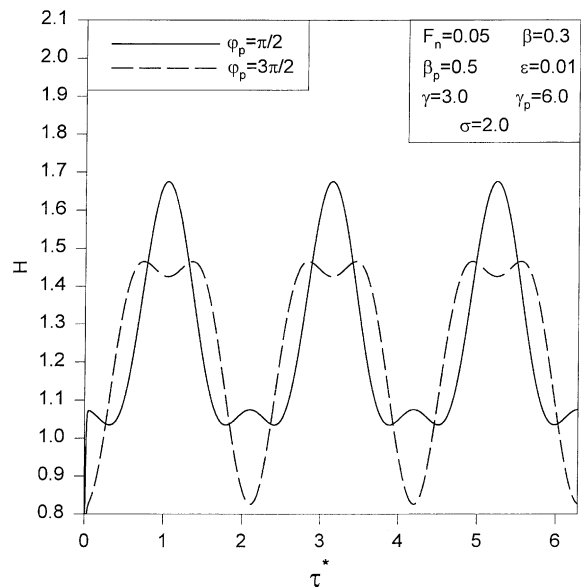


Fig. 7. Effects of ϕ_p on H .

H decreases as F_n increases. This effect can be seen from Eqs. (19) and (20) and will cause reductions in the flow and in the cooling process. However, the mean value of Π_{AVG} is always greater than zero for other values of σ which causes an increase in the mean value of H as F_n increases resulting in an increase in the mean value of the flow rate inside the thin film.

Fig. 5 shows the effects of the squeezing number σ on H . Small values of σ indicates that the thin film is having

relatively large inlet flow velocities thus it has large pressure gradients and large values of Π_0 . Accordingly, H increases as σ decreases as seen in Fig. 5. Further, it is noticed that the degree of turbulence at the upper plate increases as σ decreases. This is shown in Fig. 6. The changes in the pressure phase shift results in similar changes in the dimensionless thin film thickness phase shift as shown in Fig. 7.

4.2. Effects of pressure pulsations on heat transfer characteristics of the thin film

Figs. 8 and 9 illustrate the effects of F_n and P_S on the dimensionless mean bulk temperature θ_m and the average lower plate temperature θ_w , average of $\theta(X, 0, \tau)$, for CWT and uniform heat flux UHF conditions, respectively. As F_n increases when softer seals are used, the induced pressure forces inside the thin film due to internal pressure pulsations will increase the displacement

of the upper plate as shown before. This enables the thin film to receive larger flow rates since all the cases presented in these figures have similar values for the dimensionless pressure at the inlet. Thus, more cooling to the plates results as F_n increases resulting in a decrease in the θ_m and average θ_w values and their corresponding fluctuations for CWT and UHF conditions, respectively. The effect of the thermal squeezing parameter P_S on the cooling process is also shown in Figs. 8 and 9. It is

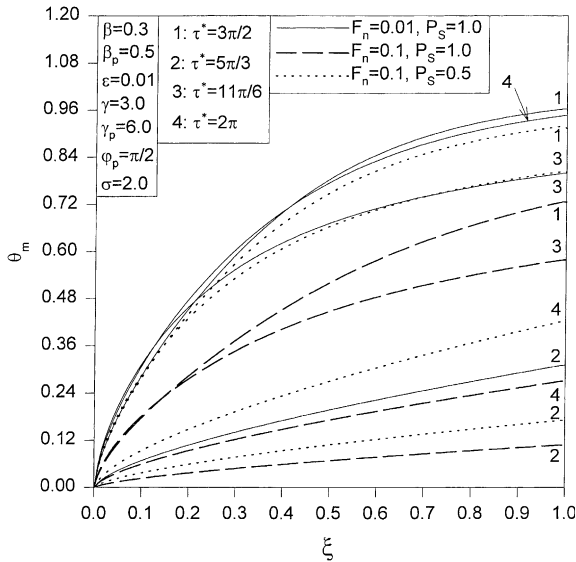


Fig. 8. Effects of F_n and P_S on θ_m (CWT).

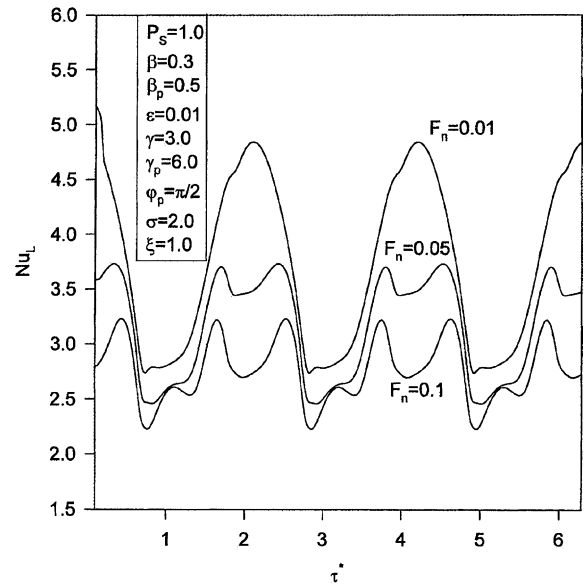


Fig. 10. Effects of F_n on Nu_L (CWT).

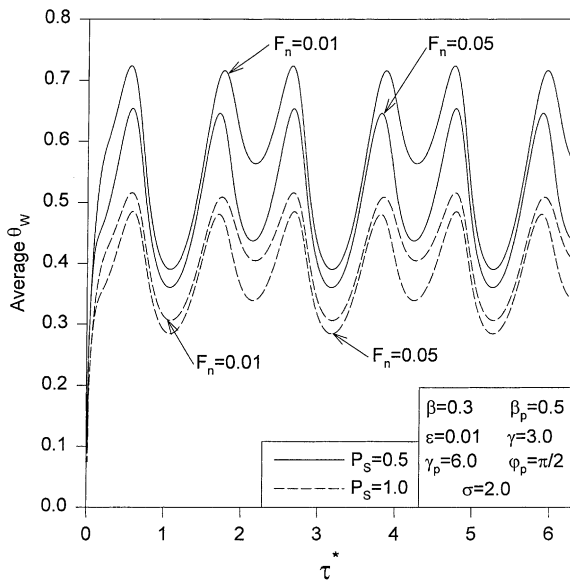


Fig. 9. Effects of F_n and P_S on θ_w (UHF).

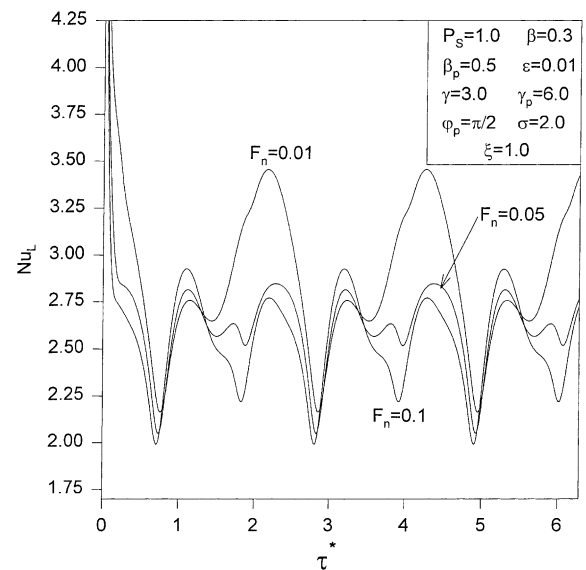


Fig. 11. Effects of F_n on Nu_L (UHF).

shown that the cooling at the plates is enhanced as P_s increases.

Figs. 10 and 11 show the effects of F_n on the Nusselt number at the lower plate Nu_L for CWT and UHF conditions, respectively. It is noticed that the irregularity in Nu_L decrease as F_n decreases. This is because the upper plate will not be affected by the turbulence in the flow if the used seals have relatively large modulus of elasticity. In other word, the induced flow due to the

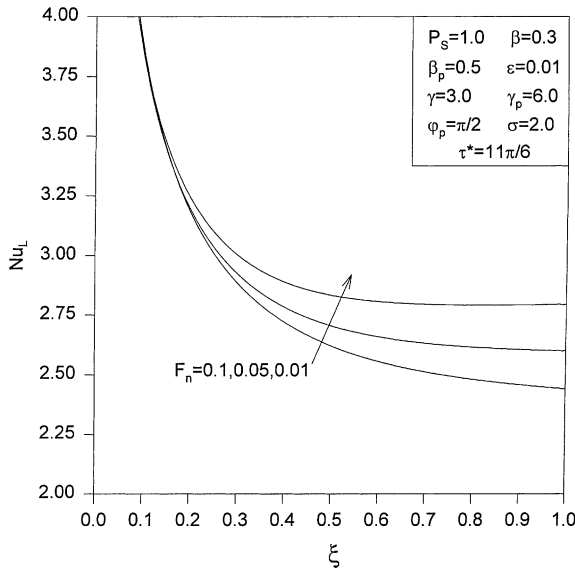


Fig. 12. Effects of F_n on the development of Nu_L (UHF).

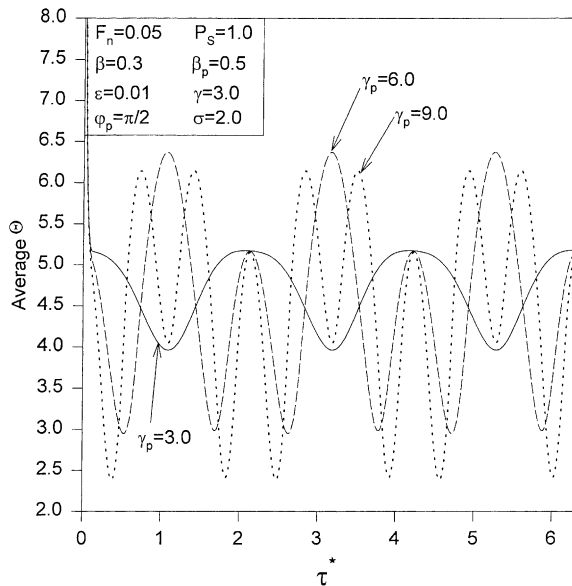


Fig. 13. Effects of γ_p on average θ (CWT).

upper plate motion is reduced as F_n decreases resulting in less disturbances to the flow inside the thin film. This can be seen in Fig. 12 for UHF conditions where Nu_L reaches a constant value at low values of F_n after a certain distance from the inlet. The values of Nu_L and the corresponding fluctuations are noticed to decrease as F_n increases.

Figs. 13 and 14 illustrate the effects of dimensionless frequency of the inlet pressure pulsations γ_p on the average dimensionless heat transferred from the plates θ and the average θ_w for CWT and UHF conditions, respectively. The figures show that the mean value of θ

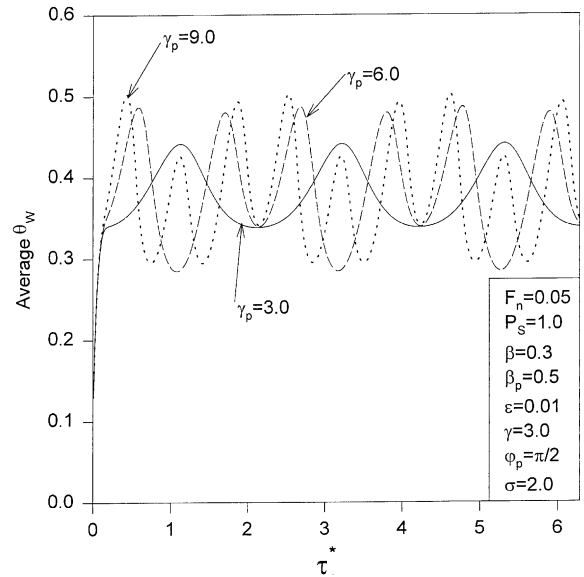


Fig. 14. Effects of γ_p on average θ_w (UHF).

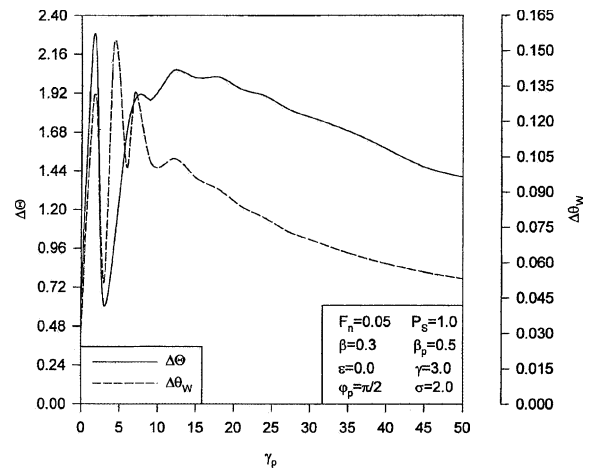


Fig. 15. Effects of γ_p on $\Delta\theta$ and $\Delta\theta_w$.

and θ_w are unaffected by γ_p and that the frequency of the average values of Θ and θ_w increase as γ_p increases. Fig. 15 describes the effects of γ_p on the fluctuation in the average Θ and θ_w , half the difference between the maximum and the minimum values of the average Θ and θ_w . The effects of γ_p on the fluctuation in the average Θ , $\Delta\Theta$, and the fluctuation in the average θ_w , $\Delta\theta_w$, are more pronounced at relatively lower values of γ_p .

5. Conclusions

Flow and heat transfer inside externally oscillatory squeezed thin film supported by soft seals in the presence of inlet internal pressure pulsations have been analyzed in this work. The governing laminar continuity, momentum and energy equations were properly non-dimensionalized and reduced to simpler forms for small Reynolds numbers. The reduced equations were solved by the ADI method. It was found that the turbulence level at the upper plate increases by increases in both the Fixation number and the frequency of the internal pressure pulsations. However, an increase in the squeezing number decreases the turbulence level at the upper plate. The fluid temperatures and the corresponding fluctuations were found to decrease when the Fixation number and the thermal squeezing parameter were increased for both CWT and UHF conditions. Finally, fluctuations in the heat transfer and the fluid temperatures are more pronounced at relatively lower frequency of internal pressure pulsations. This study can lead the way for further investigations on the effects

internal pressure pulsations on thin films having relatively large Reynolds numbers.

References

- [1] N.V. Lavrik, C.A. Tipple, M.J. Sepaniak, D. Datskos, Gold nano-structure for transduction of biomolecular interactions into micrometer scale movements, *Biomed. Microdev.* 3 (1) (2001) 35–44.
- [2] W.E. Langlois, Isothermal squeeze films, *Quart. Appl. Math.* XX (1962) 131–150.
- [3] E.A. Hamza, Unsteady flow between two disks with heat transfer in the presence of a magnetic field, *J. Phys. D: Appl. Phys.* 25 (1992) 1425–1431.
- [4] S. Bhattacharyya, A. Pal, G. Nath, Unsteady flow and heat transfer between rotating coaxial disks, *Numer. Heat Transfer, Part A* 30 (1996) 519–532.
- [5] B. Debbaut, Non-isothermal and viscoelastic effects in the squeeze flow between infinite plates, *J. Non-Newtonian Fluid Mech.* 98 (2001) 15–31.
- [6] A.-R.A. Khaled, K. Vafai, Non-isothermal characterization of thin film oscillating bearings, *Numer. Heat Transfer, Part A* 41 (2002) 451–467.
- [7] A.-R.A. Khaled, and K. Vafai, Non-Isothermal Characterization of Thin Film Oscillating Bearings in the Presence of Ultra Fine Particles, *Numerical Heat Transfer, Part A*, in press.
- [8] H.N. Hemida, M.N. Sabry, A. Abdel-Rahim, H. Mansour, Theoretical analysis of heat transfer in laminar pulsating flow, *Int. J. Heat Mass Transfer* 45 (2002) 1767–1780.
- [9] C.H. Joshi, R.D. Kamm, J.M. Drazen, A.S. Slutsky, An experimental study of gas exchange in laminar oscillatory flow, *J. Fluid Mech.* 156 (1985) 291–300.